Laboratory 7 – Matrix Operations With MATLAB
Due Week of November 21, 2011

I — Introduction:

Laboratory exercises 2, 5, and 6 dealt with measurements and how to manage “unstructured measured data”. We now turn our attention to so-called “structured” data. In particular, this laboratory introduces a very important engineering software tool for managing engineering data – MATLAB (short for MATrix LABoratory).

The main focus of this laboratory is on linear systems (systems of linear equations). You have all solved two equations with two unknowns and likely three equations with three unknowns, but it is unlikely that you have solved 20 equations with 20 unknowns. The number of equations in engineering analysis can reach this number and in some cases the number can go much higher, e.g. the number of linear (and sometimes nonlinear) equations in finite element analysis (FEA), an important engineering analytical system, can reach millions. In fact, this year you take a course in matrix methods for solving linear equations—MATH 1104 (Linear Algebra).

Linear equations occupy a special place in engineering: Ohms law ($V = IR$) is a linear equation; Newton’s second law of motion ($F = ma$) is a linear equation and many other fundamental relationships are approximately linear. These are relatively simple relationships that can be manipulated manually, however, there are many linear systems of equations where manual manipulation becomes prohibitive and software help is needed. For the most part, linear systems are set up in matrix form and MATLAB has become the software of choice, at least in recent years.

This laboratory is primarily an exercise in manipulating matrices using MATLAB. It is recommended that you read through the MATLAB support material found on WebCT and in your textbook. You can then perform the operations in the MATLAB command window, or with an m-file. Be sure to keep (save) your m-file work for your report.

We will begin by asking you to manipulate some vectors and matrices with MATLAB and end by asking you to set up and solve some related engineering problems.

II — Problem Statement:

Part 1: Vectors are a special kind of matrix that can be represented by a single column, or row of numbers (components).

1. Consider the following vectors:

$$\vec{h} = \begin{bmatrix} 7 \\ 3 \\ -5 \end{bmatrix}, \quad \vec{i} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{j} = \begin{bmatrix} 9 & 4 & -7 \end{bmatrix}, \ \vec{k} = \begin{bmatrix} 6 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \ \vec{l} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}.$$

And the following scalers: $p = 0.8$, $q = 2.2$ and $r = -0.4$.
Perform the following operations in the MATLAB command window and suggest a geometric interpretation for each allowable operation:

(a) $\vec{h} \cdot \vec{i}$

(b) $p\,\vec{k}$

(c) $\vec{l} \cdot \vec{h}$

(d) $\vec{j}^{\,T} - \vec{k}$

You should be able to do these computations from your lecture background and from the support provided in the MATLAB support material. If MATLAB returns an error, explain.

2. The CCGS Panda, a search boat for the Canadian Coast Guard, has been undergoing tests to verify the usefulness of a new navigational system. The system is supposed to guide the boat along a pre-designed course and return it to the starting location. In one particular test, the navigational system was set for the following course (each change in position is represented by a vector):

- starting at the origin (0,0), the boat traveled to a position 8 km north and 3 km east. This change in position can be represented by the vector $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$;
- the boat then traveled to a new position 6 km east and 1 km south;
- after traveling to a location 15 km west and 7 km north, the boat then went to a site 5 km east and 7 km north;
- in the final portion of this test, the boat was located 4 km east and 9 km north of the previous point.

Use MATLAB plot command to show the path of the CCGS Panda for visualization purposes: label the plot axes and make a title. Then, using MATLAB, add up all of the vectors, and draw conclusions about the results of the test: did the navigational system perform as planned? Determine the total distance traveled by the boat by finding the norm of each vector and calculating the sum.

3. Plot the equation $y = (10 + x^2 + 2x^3)/(1 + 2x^2)$ from $x = -3$ to $x = 3$ using MATLAB. Vary the number of plotted points so that the curve looks smooth.

Part 2: Use MATLAB to answer the following questions.

1. The following matrices are defined:

$$F=\begin{bmatrix}3 & 9\\ 7 & -2\\ 1 & 0\\ 6 & 8\\ 9 & 4\\ 5 & 2\end{bmatrix},\quad G=\begin{bmatrix}-9 & 2 & 8 & 5 & -1 & 2\\ 7 & 4 & -2 & 9 & 3 & 8\end{bmatrix},\quad Q=\begin{bmatrix}4 & -7 & 2\\ 1 & 0 & 8\\ -5 & 3 & 5\end{bmatrix},\quad U=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

   (a) Multiply F and G and then multiply the result by the scalar r defined previously;

   (b) Transpose G, then perform the matrix operation $H = F - G^T$;

   (c) Sort and then plot H, using the vector $[1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6]^t$ for the x indices.

   (d) Multiply U and G ;

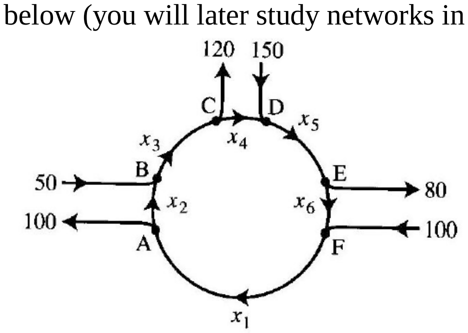
   (e) Multiply F and U ;

   (f) Multiply Q and h, which was defined previously;

   (g) Perform the following operation: $(q \cdot G) + (p \cdot F^t)$.

2. A linear transformation $T(\vec{x})$ maps the "original" vector $\vec{x}$ to its "image" $\vec{b}$ via the following matrix equation: $A\vec{x} = \vec{b}$ where A is called the "standard matrix" (you will later study linear transformations in MATH 1104). Consider a particular linear transformation $A=\begin{bmatrix}9 & 2 & 5\\ 1 & 7 & 6\\ 2 & 3 & 1\end{bmatrix}$. If the image produced with this transformation is $\vec{b}=\begin{bmatrix}5\\ 1\\ 3\end{bmatrix}$, then find the original vector $\vec{x}$?

3. Busy intersections in Europe are often constructed as one-way "roundabouts", such as the traffic network in England displayed below (you will later study networks in MATH 1104):



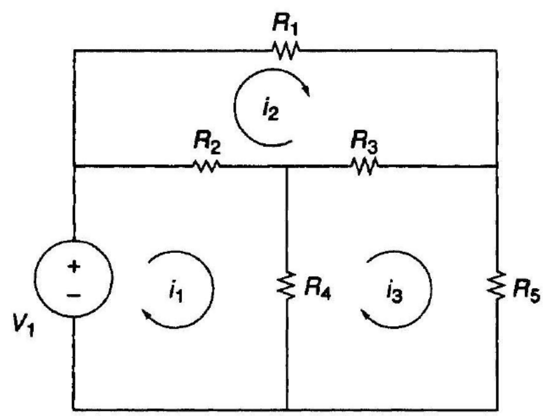
   Assuming that the traffic must travel in the directions shown, to find the unknown flow rates in cars/minute, solve the following system of equations:

$$
\begin{cases}
x_1 - x_2 = 100 \\
x_2 - x_3 = -50 \\
x_3 - x_4 = 120 \\
x_4 - x_5 = -150 \\
x_5 - x_6 = 80 \\
-x_1 + x_6 = -100
\end{cases}
$$

Write these equations in matrix form, $B\,\vec{x} - \vec{c}$, and solve using MATLAB. If MATLAB does not return an answer, explain. Is it possible to solve this system at all? Use the "pseudoinverse" function `pinv ( )` to obtain a solution. Check the solution by calculating $B\,\vec{x} - \vec{c}$

4. A steam plant burns two types of coal: anthracite (A) and bituminous (B). For each tonne of A burned, the plant produces 27.6 million Btu of heat, 3 100 grams (g) of sulphur dioxide, and 240 g of particulate matter (solid-particle pollutants). For each tonne of B burned, the plant produces 30.2 million Btu, 6 400 g of sulphur dioxide, and 350 g of particulate matter. Over a certain time period, the steam plant produced 162 million Btu of heat, 23 610 g of sulphur dioxide, and 1 566 g of particulate matter. Determine how many tonnes of each type of coal the steam plant must have burned.

5. An essential component of the new intelligent disco light system you are currently building for your next party includes the following electrical circuit which contains a single voltage source and five resistors:



Using Kirchhoff's Voltage Law and Ohm's Law, the following set of equations defining the currents in this circuit is obtained:

$$
\begin{aligned}
-V_1 + R_2(i_1 - i_2) + R_4(i_1 - i_3) &= 0 \\
R_1 i_2 + R_3(i_2 - i_3) + R_2(i_2 - i_1) &= 0 \\
R_3(i_3 - i_2) + R_5 i_3 + R_4(i_3 - i_1) &= 0
\end{aligned}
$$

If $R_1 = 7\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 7\ \Omega$, $R_5 = 2\ \Omega$, and $V_1 = 8$ V, compute the currents.

Sketch this circuit diagram using IntelliCAD. Use the circuit sketching blocks provided under the "Support Material" on the WebCT to assist you. All circuit components should be labelled with their respective numerical values (in ohms, amps or volts).

6. Make a 3 by 3 matix with the first row equal to the first three numbers of your student card, and the second row equal to the next three numbers, and the third row equal to the last three numbers of your student card; call this matrix A. Perform the following calculation and report the value c:

   $c=b*A*b^t$

   where $b = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

III — Steps and Calculations:

Using MATLAB and its functions, perform all the necessary steps to obtain the answers to each of the questions and problems presented above. Use IntelliCAD to sketch the circuit in Part 2 (step 5) to provide visual support to the solutions.

IV — Report Requirements:

- Using the guidelines presented in Laboratory 1, produce a formal laboratory report that summarizes your findings.
- It is apparent that this is a somewhat different laboratory exercise in the sense that it is more or less a series of problems. Nonetheless you should be able to identify a central focus (central objective) to use as guidance for writing your report.
- State briefly the results to all of the problems posed. Discuss the significance of the results in each case.
- Show your MATLAB code for each problem in an appendix(ces).
- Include a short discussion of the advantages and disadvantages of software such as MATLAB for dealing with vectors and matrices.

V — Submission and Timing:

Your report is to be submitted to the Teaching Assistant within the first 30 minutes of your next laboratory period (week of November 21, 2011). **<u>LATE SUBMISSIONS WILL NOT BE ACCEPTED.</u>**

VI — Marking:

Laboratory submissions will be marked on a 10-point scale: 9-10 (excellent); 7-8 (good); 5-6 (marginal); less than 5 (fail). **Be sure that you are familiar with the University's policy on plagiarism and academic integrity. Your instructors are obligated to report all suspected violations to the Associate Dean's office for investigation.**